**Indian Statistical Institute, Bangalore**

B.Math (Hons.) III Year / M.Math II Year, Second Semester

Semestral Examination, Back Paper

Combinatorics and Graph Theory

Time: 3 hours May 00, 2011 Instructor: B.Bagchi

Maximum marks: 100

1. If $V$ is an $(n+1)$- dimensional vector space over $\mathbb{F}_q$ then let $X$ be the incidence system whose points and blocks are the 1- and $n$- dimensional linear subspaces of $V$, and incidence is set inclusion ($\subseteq$). Show that $X$ is a 2- design and compute its parameters. [20]

2. Let $V$ be the collection of all 4- subsets of a 12- set. Let $G$ be the graph with vertex set $V$ such that $A, B \in V$ are adjacent in $G$ iff $\sharp(A \cap B) = 0$ or 2. Show that $G$ is a strongly regular graph and compute its eigenvalues and their multiplicities. [30]

3. Let $G_n$ be the graph with vertex set $S_n$ such that $\alpha, \beta \varepsilon S_n$ are adjacent in $G$ iff $\alpha \circ \beta^{-1}$ fixes at most one symbol. Show that the clique number $\omega(G_n)$ of $G_n$ is at most $n(n-1)$. If, for some $n, \omega(G_n) = n(n-1)$, then show that there is a projective plane of order $n$. [10+10 = 20]

4. Prove the existence and uniqueness of the projective plane of order 4. [20]

5. Construct an explicit Hadamard matrix of order 12. (It is not enough to give some kind of formula for its entries). [10]